Loan Amortization and Yield Calculation, An Example

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Consider the following note

- note amount : $25
- grade : A1
- interest rate: 5.32%
- term : 36 months

Table 1 lists its amortization schedule and the Yield to Maturity (YTM) if bought at par (i.e., price = outstanding principal) at each particular month and held until maturity.

Suppose user A bought the note fresh at issue.

Q1. What yield would A get if A held the note until maturity? (i.e., the YTM at issue)

A1. **4.43%**. We can simply look it up from table 1: cell of Month 1 and YTM_w_fee.

Scenario 1 (Braden's Scenario)

Suppose A only held it for 18 months, and at the beginning of month 19, A sold the note to B at a price that gives B a YTM of 4.43% (the implicit assumption here is that B holds the note until its maturity).

Q2. What's the transaction price?

A2. Let it be P, then by the price/yield formula, we have

$P = \frac{C}{1+r} + ... + \frac{C}{(1+r)^{18}}$, where C is net_cash (0.743), and $r = \frac{YTM_B}{12} = \frac{0.0443}{12} = 0.003691667$. Solving for P, we get **$12.91**.

Q3. What yield would A get at such a transaction price?

A3. Let it be z, then its monthly yield is $x = \frac{z}{12}$, and by the price/yield formula, we have

$25 = \frac{C}{1+x} + ... + \frac{C}{(1+x)^{18}} + \frac{P}{(1+x)^{18}}$, where C is net_cash (0.743) and P is $12.91. Solving for x and hence z, we get **4.43%**.

Scenario 2

Suppose user A bought the note fresh at issue and held it for 18 months. Suppose at the beginning of 19th month, A sold the note to B at par for $12.997. Suppose B held the note for the remaining 18 month until its maturity.

Q1. What yield did B get?

A1. **3.61%**. We can simply look it up from table 1: Cell of Month 19 and YTM_w_fee.

Q2. What yield did A get?

A2. Let it be z, then its monthly yield is $x = \frac{z}{12}$, and by the price/yield formula, we have

$25 = \frac{C}{1+x} + ... + \frac{C}{(1+x)^{18}} + \frac{12.997}{(1+x)^{18}}$, where C is net_cash (0.743). Solving for x and hence z, we get **4.7%**.

Table 1: Amortization and YTMs

month	outstanding_principal	principal	interest	fee	my_interest	payment	net_cash	YTM_no_fee	YTM_w_fee
1	25.000	0.642	0.111	0.01	0.101	0.753	0.743	5.32	4.43
2	24.358	0.645	0.108	0.01	0.098	0.753	0.743	5.32	4.40
3	23.713	0.648	0.105	0.01	0.095	0.753	0.743	5.32	4.38
4	23.065	0.651	0.102	0.01	0.092	0.753	0.743	5.32	4.35
5	22.415	0.653	0.099	0.01	0.089	0.753	0.743	5.32	4.32
6	21.761	0.656	0.096	0.01	0.086	0.753	0.743	5.32	4.29
7	21.105	0.659	0.094	0.01	0.084	0.753	0.743	5.32	4.26
8	20.446	0.662	0.091	0.01	0.081	0.753	0.743	5.32	4.23
9	19.783	0.665	0.088	0.01	0.078	0.753	0.743	5.32	4.19
10	19.118	0.668	0.085	0.01	0.075	0.753	0.743	5.32	4.15
11	18.450	0.671	0.082	0.01	0.072	0.753	0.743	5.32	4.11
12	17.779	0.674	0.079	0.01	0.069	0.753	0.743	5.32	4.06
13	17.105	0.677	0.076	0.01	0.066	0.753	0.743	5.32	4.01
14	16.428	0.680	0.073	0.01	0.063	0.753	0.743	5.32	3.96
15	15.748	0.683	0.070	0.01	0.060	0.753	0.743	5.32	3.90
16	15.065	0.686	0.067	0.01	0.057	0.753	0.743	5.32	3.84
17	14.379	0.689	0.064	0.01	0.054	0.753	0.743	5.32	3.77
18	13.690	0.692	0.061	0.01	0.051	0.753	0.743	5.32	3.69
19	12.997	0.695	0.058	0.01	0.048	0.753	0.743	5.32	3.61
20	12.302	0.698	0.055	0.01	0.045	0.753	0.743	5.32	3.51
21	11.604	0.701	0.051	0.01	0.041	0.753	0.743	5.32	3.41
22	10.902	0.705	0.048	0.01	0.038	0.753	0.743	5.32	3.29
23	10.198	0.708	0.045	0.01	0.035	0.753	0.743	5.32	3.16
24	9.490	0.711	0.042	0.01	0.032	0.753	0.743	5.32	3.00
25	8.779	0.714	0.039	0.01	0.029	0.753	0.743	5.32	2.83
26	8.065	0.717	0.036	0.01	0.026	0.753	0.743	5.32	2.62
27	7.348	0.720	0.033	0.01	0.023	0.753	0.743	5.32	2.38
28	6.628	0.723	0.029	0.01	0.019	0.753	0.743	5.32	2.09
29	5.905	0.727	0.026	0.01	0.016	0.753	0.743	5.32	1.73
30	5.178	0.730	0.023	0.01	0.013	0.753	0.743	5.32	1.29
31	4.448	0.733	0.020	0.01	0.010	0.753	0.743	5.32	0.71
32	3.715	0.736	0.016	0.01	0.006	0.753	0.743	5.32	-0.05
33	2.978	0.740	0.013	0.01	0.003	0.753	0.743	5.32	-1.12
34	2.239	0.743	0.010	0.01	0.000	0.753	0.743	5.32	-2.71
35	1.496	0.746	0.007	0.01	-0.003	0.753	0.743	5.32	-5.38
36	0.750	0.750	0.003	0.01	-0.007	0.753	0.743	5.32	-10.69
37	0.000	NA	NA	NA	NA	NA	NA	0.00	0.00

How to ensure both A and B to get the same yield?

Let their common yield be Y and the transaction price be P. By price/YTM formula, we have the following 2 equations and 2 unknowns.

- $25 = \frac{C}{1+x} + ... + \frac{C}{(1+x)^{18}} + \frac{P}{(1+x)^{18}}$
- $P = \frac{C}{1+x} + ... + \frac{C}{(1+x)^{18}}$

where C is net_cash (0.743) and $x = \frac{Y}{12}$.

Solving for both x and P together, we get a common yield of **4.43%** and a transaction price of **$12.91**.

Note: this common yield is the same as the YTM one would get if one buys the note fresh at issue and holds it until maturity.

What if transaction happens before note's mid life?

Say sale/purchase happens at month 5, then we have a common yield of **4.43%** and a transaction price of **$21.72**.

What if transaction happens after note's mid life?

Say sale/purchase happens at month 25, then we have a common yield of **4.43%** and a transaction price of **$7.99**.

Conclusion

The above calculation seems to indicate that we can use the YTM at issue (the yield one would get if one buys the note fresh and holds it till maturity) as the common yield and calculate the transaction price.